Thinking About Spatial Comparisons

The passes between these Asian mountains are higher than the top of Pike's Peak in Colorado.

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This chapter is about organizing and representing geographic information by comparing places. Comparison is an obvious and fairly easy way to limit the amount of factual memorization. All you have to do is learn a few key facts about conditions or connections at one place, and then you can use that information as a kind of benchmark for comparing other places. Moreover,

> "[T]he process of comparison, viewed as a process of structural alignment, constitutes an important route towards abstract conceptual understanding" (Gentner and Namy 1999).

A thematic map, from this perspective, is basically a device for displaying facts about places in a way that makes comparison easy. To facilitate comparison, mapmakers use a variety of graphic devices, such as gray-scale shading, color, dot densities, line widths, scaled symbols, or shape distortions.

Discussion of comparison is complicated, because the term includes two different kinds of activities:

A **qualitative** comparison looks at differences in kind, not in quantity. Examples include political jurisdiction (e.g., Spain or France), geologic structure (granite or sandstone), language (Arabic or Swahili), or religious tradition (Sunni or Shiite).

A **quantitative** comparison tries to decide which of two places has a greater quantity or higher measurement of something, such as temperature, wheat yield, household income, average life expectancy, or Republican vote in the last election.

Both kinds of comparison start by using the same basic brain structures to perceive or recall something about conditions in one place, store that information in some kind of temporary buffer or working memory, and get information about the other place. At that point, however, qualitative and quantitative processes seem to diverge.

A qualitative comparison relies primarily on verbal memory, putting the information into categories and deciding whether the places are similar or different (Namy and Gentner 2002). Cartographers refer to this kind of information as being "nominally scaled." They have a number of conventional ways to show nominal information – dissimilar point symbols, color hues, and so forth. The rules for selecting nominal symbols are usually described in terms of what they should NOT be – they should not be different in size, color intensity, brightness, or textural complexity. In other words, they should be "non-quantitative."

By contrast, the conventional way to depict quantitative information is by using symbols that vary in size, density, brightness, or color intensity. Most cartographers follow relatively simple rules of thumb, such as "wider is more" and "darker is more." During the 1970s and 80s, a number of cartographers tried to make a more rigorous formulation of these rules about symbol scaling. This research was prompted by the observation that a map symbol that is actually three times as large as another is perceived by some people as barely twice as large.

In short, visual perception of symbol sizes, line widths, and color intensities seems to be non-linear, and some people did experiments to quantify the non-linearity (widely cited examples include Jenks and Knos 1961; Flannery 1971; Dobson 1974; Provin 1977; Gilmartin 1981; Slocum 1982).

By the late 1980s, the pace of empirical investigation of this topic had slowed considerably. Ironically, as interest among geographers waned, neuroscientists began to explore the topic more intensively, aided by new technologies such as functional magnetic resonance imaging (fMRI), positron emission tomography (PET), and transcranial magnetic stimulation (TCMS). What is especially intriguing (and gratifying) is how the recent brain-scanning research seems to suggest that many "map languages" are actually quite solidly grounded in fundamental neurologic structures (see, for example, Pinel et al. 2004; Piazza et al. 2007; Cohen Kadosh et al. 2008).

Quick Review of Research on Thinking about Spatial Comparisons

An early highlight in this subfield of neuroscience is a famous study done in the late 1930s. This study was based on careful observation of an individual who had the kind of brain damage that drastically interfered with control over finger movement. One "side effect" of this brain lesion was a nearly complete loss of ability to do many kinds of mathematical calculation and comparison (Gerstmann 1940). The investigators inferred that the brain learns to count by manipulating fingers or mental images of fingers, and other aspects of mathematical thinking rely on these counting mechanisms.

For several decades, the "Gerstmann syndrome" was viewed primarily as a curiosity, a somewhat surprising consequence of partly overlapping brain networks. Indeed, its major role in psychology classrooms seemed to be as an anecdote that teachers could deploy when they observed a lag in student interest. A few researchers looked at how rapidly people could do numerical tasks under different conditions. For example, some studies in the 1960s tried to assess how much a finger-movement task would interfere with numerical processing in healthy individuals (e.g., Moyer and Landauer 67). Others tracked the life-cycle changes in competence for various numerical tasks. Some of those studies hinted at a cognitive difference between *subitizing* (the seemingly instantaneous mental processing of small numbers of objects) and *counting* (the slower mental comparison of larger numbers of objects (Trick et al. 1996). Good cartographers made use of this information to aid map comprehension, for example by choosing representation ratios for a dot map so that clusters would generally have no more than five or six dots.

Modern brain scanning has found that both subitizing and counting seem to involve brain tissue in a specific pair of folds in the brain surface. You have one of those "trenches," called the intraparietal sulcus (IPS), on each side of your head, above and behind your ear. In most people, however, the left IPS is more important for counting, whereas the right IPS is more engaged during subitizing tasks (for different views on this, compare Reynvoet and Ratinckx 2004 and Cohen Kadosh et al. 2005; for new looks at the original Gerstmann findings, see Kaufmann et al. 2006 and Rusconi et al. 2008).

Late in the 20th century, the floodgates opened on research about mental comparison. Hundreds of studies were done in a number of labs in the United States, Europe, and Asia. The main conclusion of this flurry of research is simple: the "comparison machine" in the human brain appears to be a logarithmically-scaled number-line that is used to make quick comparisons of verbal numbers, number symbols, object sizes and numerosities, dot densities, texture complexities, color intensities, color hues, sound volumes, and perhaps a number of other aspects of the environment around a person (for a review, see Piazza et al 2006 and 2007).

It follows that the brain structures in the IPS are what a person uses to compare many of the symbols that cartographers have developed to display quantitative information on a map. Moreover, the logarithmic nature of the mental number line helps explain why human perception of map symbols such as graduated circles or shades of gray is not linear (Flannery 1971 and the list of studies in the introduction to this essay).

This perceptual nonlinearity has implications for many aspects of map symbology, including color selection and scaling of point and line symbols. Unfortunately, those inferences are not consistently incorporated into the default color palettes and symbol panels of various GIS programs, mapping software, and web-mapping sites. Caveat emptor!

The research story about comparison does not stop here, however. The lines of evidence appear to curve around almost to the beginning of this series of essays. Research now seems to suggest that the human mental concept of location is actually derived from a quantitative comparison of the effort of overcoming distance between places. In effect, our ideas about location may really be conclusions that we draw after mentally going through a series of quantitative questions – “how far,” “how fast,” and “how long” – and comparing the answers (Walsh 2003).

So where does this leave us as designers of educational materials? It seems that students should “encounter” activities that force them to wrestle with three issues, not necessarily in this order:

1) how to make a fair comparison of places. This can be harder than it sounds, because it usually requires knowing how to adjust (normalize) data when comparing two places that are also greatly dissimilar on some other relevant measure. For example, one cannot draw valid inferences about the production of food in two states without taking into account the fact that the two states might have different areas or populations (see the CD unit about Capital Gains in California). One solution is to “normalize the data” by dividing the total food production by the population in each state, in order to get a quantitative measure of food production per person. This procedure is so important in geographic analysis that nearly every kind of GIS software allows users to make calculations like these prior to mapping. A skillful map reader, however, still must be able to figure out what variables were combined in order to produce the ratios that are depicted on a given map.

2) how to use a map or other geographic representation as a source of information for comparing places. This also can be more difficult than it sounds, because the human visual system does not “capture” things like color intensity, symbol size, or numerosity in a linear way (e.g., where twice as large or twice as many dots per square inch is perceived as exactly twice as much). Some cartographers are aware of this fact and others, alas, are not (and still others choose to ignore it in order to manipulate the map reader’s perceptions). Unfortunately, that issue is further complicated by the fact that some online and standalone GIS or mapping software allows the user to adjust for perceptual nonlinearity, while other packages do not. A skillful map reader, therefore, must be able to recognize whether map-makers have considered perceptual nonlinearity in their selection of symbols.

3) how to make valid inferences about unmeasured or unreported variables, when all you have is a comparison of some variables that could be measured and reported. This does not sound easy, but it can be even more difficult than it sounds, because the suite of variables that are relevant for a particular comparison is itself a geographic phenomenon – it can change from place to place. Fortunately, some of the other modes of spatial thinking (especially those dealing with regions, spatial transitions, spatial analogies, and spatial associations) can help people decide what information is likely to be relevant. For that reason, we will focus the activities in this chapter on the first two questions and leave this one until later in the book.

Geographic Comparison - which places are most alike?

Instructions: These numbers describe some characteristics of New York City, New York State, and three places where many New Yorkers have relatives: the Dominican Republic, Haiti, and Puerto Rico. On each row,
1. put a star next to the highest number,
2. draw a line underneath the lowest number,
3. circle the two (2) numbers that are most alike. Then write a sentence to describe the similarity.
Here is an example: New York State and the Dominican Republic are about the same size; New York City is much smaller, and Haiti and Puerto Rico are in between.

Topic	New York City	New York State	Dominican Republic	Haiti	Puerto Rico
General description	Part of Long Island, two .large and many small islands, plus part of a continent	Part of a continent plus some islands (including all of New York City)	Part of an island (Hispaniola) plus some small islands	Part of an island (Hispaniola) plus some small islands	A medium-sized island plus some small islands
Area in square miles	305	55,000	48,000	12,000	3,500
Highest place in feet above sea level	400	5,300	10,400	7,500	4,400
Population in millions	8	19	9	9	4
Population density (people per square mile)	26,000	400	225	900	1,100
Number of children less than 14 years old in each 100 people	15	19	32	42	21
Number of people more than 65 years old in each 100 people	15	13	6	4	14
Yearly income per person in thousands of dollars	44	40	7	3	20
Number of farm workers in each 100 workers	0	1	21	66	7

Sample dialogs from two teachers trying to explain why one should "normalize" GIS data.

T: Here is a table of data about crime in [state]. What city has the biggest crime risk?

S1: [city name] has the most in almost every category – murders, burglaries, auto thefts.

T: True, but does that make it the biggest risk? Think about it from the point of view of an individual person. In which city do you have the highest chance of getting mugged?

S2: I see. [Name] also has the biggest population. So your own personal chance of getting mugged is actually higher in [city]. They have almost as many muggings and a much lower population.

T: That's really good. So you can't just compare total number of crimes in order to figure out the risk for an individual. What should we do to our crime data in order to make a fair comparison of two places in terms of their crime risk?

S2: How about divide the number of crimes in each city by the total population of that city?

T: right. In order to make a fair comparison, we have to divide the total number of crimes in a city by its population, in order to find out the number of crimes per million people. That's called normalizing the data. And the way we do that in a GIS is by applying a formula to the data table, like this . . . .

T: Look at this table of crime data. How can we make a fair comparison of crime risk in [city] and [city]? S: [awkward silence]

T: Well, if a big city has ten murders a year, is that a bigger risk for an individual than in a small city?
S: No. With more people around, you have less chance of being one of the ten.

T: That's really good. To compare risk for an individual, you need to know the population as well as the number of crimes. Now, before we see how to handle that in a GIS, let's explore the basic idea. Suppose one country has a million acres of forest and another one has ten million. What other fact could complicate that comparison?

S: Well, if the first country was tiny, and the second one huge . . .

T: Right. To see the importance of forest land, we have to consider the total area of the country. There is a big point here, so I'm going to say it slowly: to make a fair comparison of places, you can't just look at total numbers of something. You have to divide those numbers by whatever other information is important. To figure crime risk for a person, you divide the number of crimes by total <u>population</u>. To figure importance of forest, you divide forest land by total <u>area</u>.

These dialogs illustrate that the difference between a lesson that reinforces a specific mode of spatial thinking and one that does not is often quite subtle – in short, "the devil is in the details," and the details are cumulative

The two dialogs are the same length (230 words). Both teachers get at the main point, but the second one makes sure students can generalize the principle before they learn the formula. Otherwise, students may just remember the procedure ("divide each number by the total population") and misapply it in situations where the divisor should be something else, like area, total production, exports, etc.

[layout idea – the two dialogs could be highlighted/differentiated with different background colors? that could be a consistent marker in every chapter, helping readers stay oriented]

Additional student activities that involve thinking about spatial comparisons

A. Comparing two blocks on a street according to some practical measure

1. Imagine you work for a window washing (or lawn mowing) service, and your job is to estimate how much to charge for doing the work on two different blocks
2. Go out and count or estimate the number and size of windows on the buildings (or the total size of the lawns) on several different blocks (math extension: figure an average?)

CAUTION: this field activity can be perceived as busywork unless you take care to

- position the activity as something people have to do in everyday life (cite plenty of locally relevant examples, such as painters, brick-repairers, cable TV installers)
- emphasize that the goal is comparison – students should seek a method of fast estimation that allows accurate comparison, not precise measurement
- encourage students to reflect on the process they used to derive their measure, and focus the debriefing discussion on that process, not the accuracy of their measures

B. Comparing a foreign country with a similar-sized state or group of states

1. Select a foreign country (or note one that was in the newspaper recently)
2. Look up its area and population in the CIA Factbook, an online source, or the tables we made from the Factbook and put on the CD
3. Find a state or group of states that have roughly the same area
4. Write an explicitly comparative sentence, like "Bangladesh has almost exactly the same area as Iowa, but it has nearly 50 times as many people"

If each student picks a different country and prepares a poster, one valuable take-away message from comparing the posters is the realization that most parts of the world are significantly more crowded than the United States. *This is a point worth emphasizing in any geography class!*
Variation: compare countries and state according to other measures, such as income, life expectancy, internet usage; students should think about what to use as a "base" instead of area.

C. Make a magazine/TV map with unusual symbols, or a physical map with small artifacts

1. Find an interesting data set, such as mountain height, city population, flood depth
2. "Invent" an attention-grabbing way to display the data – e.g. by drawing the appropriate number of small Empire State Buildings to represent the height of mountains, or stacking pennies on a desk map of U.S. cities (1 penny can show one million people)

D. Make a map (or design a GIS display) in order to persuade

1. Find an interesting data set, such as city size, average income, unemployment, life expectancy
2. Make a map using the default settings for symbol scaling (for a point-symbol map) or selection of data-category boundaries (natural breaks, quantiles, etc. on a choropleth map).
3. Adjust the symbol scaling or category boundaries to emphasize each extreme:
   - that most of the country or state is quite similar, with only a few exceptional places
   - that there are great differences within the country

It is very important that students do this at least once in their middle-school years, so that they realize the extent to which a map can be changed in appearance (while remaining technically "accurate") by the choices a cartographer makes when deciding about such "boring" details as symbol scaling and data-category breakpoints

E. Calculate (or estimate) an Index of Local Importance (see CD unit for details)

1. Select two areas of interest, and some measures to use in comparing them
2. Express the comparison in terms of regional or world averages, rather than simply between two places. "The population density (people per square mile) of New York is five times the national average; Texas' population density is equal to the national average; Oklahoma has only half as many people per square mile as the U.S. as a whole"

Detailed review of research about spatial comparisons

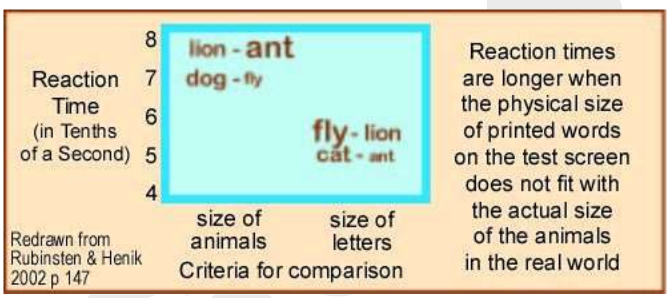
Serious attempts to probe the brain processes involved in comparison of physical size, color, brightness, or number began in the middle of the 20th century (e.g. Moyer and Landauer 1967). The early 1990s saw some of the first empirical studies that

- described a "mental number-line" (e.g. Dehaene et al. 1993),
- explored how cerebral lesions could disrupt the number-line (Cohen and Dehaene 1996), and
- speculated about a possible neuronal structure the might perform comparisons of dot numerosities as well as discrete numbers (Dehaene and Changeux 1993).

Ten years later, the same author did some of the first brain-scanning studies, which localized the mental number-line in the brain region called the intraparietal sulcus (Simon et al 2002; Dehaene et al. 2003; see also Rivera et al. 2005; Venkatraman et al. 2005; Ansari et al. 2006; see the general review in Wood et al. 2008).

Within a few years, the findings of those fMRI studies had been verified by several completely independent technologies. For example, one study described a genetic alteration of the IPS that had adverse effects on numerical abilities (Molko et al. 2005). Other researchers showed that a transcranial magnetic pulse over the same part of the brain could distort the mental number line (Sandrini et al. 2004; Goebel et al. 2006). This distortion, however, appears to be different for male and female subjects, in some cases producing diametrically opposite results (Knops et al. 2006).

Meanwhile, a swarm of behavioral studies were exploring aspects of the mental number line. Some, for example, explored the possibility that unitary and decade numbers ("ones" and "tens") were somehow encoded differently (Nuerk et al. 2001). One particularly elegant experiment noted that reaction times tend to increase if the names of animals are presented in font sizes that do not match the relative sizes of the animals in nature, e.g. by asking subjects whether an "ant," written in large type, is larger than a "lion" printed in smaller type (Rubinstein and Henik 2002; for a related study involving the sizes of printed numbers, see Kaufmann et al. 2005).



Other empirical studies

- showed that the ability to make comparisons based on size or numerosity were common among animals of many species (e.g. fish in Agrillo et al. 2011; birds in Rugani et al. 2015)

concluded that brain mechanisms for estimating quantities were similar in 5-year-olds and adults (Temple and Posner 1998),

- explored developmental changes in some of the details of size and numerosity comparisons (Paik and Mix 2008; Odic et al. 2013)
- straddled the border between psychology and vision science by exploring why people often show rightward eye movements when they are thinking of larger numbers (Schwarz and Keus 2004; Knops et al. 2009),
- noted that patterns of brain activity are similar when people recite verbal numbers and other ordered words, such as the letters of the alphabet (Zhou et al. 2006),

- verified that the mental number line seems to go from left to right, regardless of the direction of writing in the primary language of the subject (Zebian 2005; compare Gelman and Butterworth 2006; Venkatraman et al. 2006; Shaki et al. 2008, 2009),
- reported the apparent existence of a separate vertical number line, with low numbers at the bottom and higher near the top (Ito and Hatta 2004). A later study suggested that "one relies on a visual-spatial code and the other on a verbal code" (Cohen Kadosh et al. 2007),
- noted that numbers and spatial relations were linked by the use of some common brain circuits (Hubbard et al. 2005). A later study showed that this interaction occurred only when subjects had enough time for an expectancy to develop for the number cue (Stoianov et al. 2008),
- noted that the mental number line seemed to recalibrate itself when a subject is told that a particular array of dots contained a particular number, even if that number was inaccurate (Izard and Dehaene 2007; see also Stewart et al. 2002 and Grabner et al. 2009), and, to come full circle from the beginning of this list,
- added complications due to interference, time conventions, other variables (van Opstalk et al. 2008; Cappelletti et al. 2011; Núñez et al. 2011; Pfister et al. 2013; and many others)
- explored various ways in which perceptual tasks could interfere with quantity estimation of numerosity in pre-schoolers (Rousselle and Noel 2008)
- looked at the possibility of learning transfer following electromagnetic stimulation of parietal cortex in the brain (Cappelletti et al. 2013)

More recent studies show that the IPS "number line in the brain" also seems to get involved when people are asked to judge brightness (Cohen Kadosh et al. 2008). It was already known that the brightest part of a scene tends to calibrate the perceived gray scale (Gilchrist et al. 1999), but new evidence suggests that prior perception of low or high numbers can also bias judgments of gray-scale brightness (Nicholls and Loftus 2008).

All of these findings have implications for the design of maps, especially choropleth, scaled-symbol, and dot maps. In this context, it is worth noting that the ability of a map legend to influence initial perception of map symbols as well as subsequent decoding of map symbology has long been suspected by cartographers such as Judy Olson, who argued for the need to pay attention to both the visual and the psychological aspects of map reading (Olson 1979). The take-home message for cartographers is that the display solutions which computer programmers find convenient to implement in their software may not always be the optimal ones for cartographic design.

Interesting offshoots of this research have examined
- use of quantitative comparison by isolated, non-literate groups of people (Pica et al. 2004),
- other effects of verbal language on magnitude comparison (Nuerk et al. 2005),
- uneven deterioration of comparative ability due to dementia (Jefferies et al. 2005),
- effects of traumatic brain injury on size or number comparison (Revkin et al. 2008),
- links between quantitative comparison, spatial perception, and music (Lidji et al. 2007),
- ability of some animals to compare population densities and adjust behavior (Ansley et al. 2009; for studies of subitizing by the birds and the bees (sorry about that!), see Holden 2009 – one conclusion is that even insects can learn to recognize and fly toward flowerpots that have 2, 3, or 4 flowers in them, regardless of flower color), and even
- whether spatial comparison can clarify the effect of air pressure on how far baseballs fly in different ballparks (Chambers et al. 2006)!

Finally, some recent studies used ingenious methods to focus on how the spatial character of the mental number line might be important in understanding how children build number representation during early school years (Bachot et al 2005; Ansari and Dhital 2006; Boyer and Levine 2012). A later study explored the implications of this research for mathematics education, and concluded that

> "while children of all ages [6,7,9] forecasted linear growth adequately, exponential growth was also estimated remarkably well. Surprisingly, kindergartners and third graders showed similar high achievement . . . whereas first graders performed significantly worse. We concluded that primary knowledge of both linearity and non-linearity exists even in kindergartners" (Ebersbach et al. 2008).

A later study by the same author noted that young children performed better than all other ages, including adults, in some volume-estimation tasks (Ebersbach 2009). Another study concluded that a "[spatial] number board game significantly improved children's performance in all posttest measures and facilitated a shift from a logarithmic to a linear representation of numerical magnitude, emphasizing the importance of spatial cues in estimation" (Whyte and Bull 2008).

We suspect that these are just the first of many studies that will explore the effects of the "forced shift" from intuitive (logarithmic) size estimation to linear mathematical thinking in early grades. Moreover, we think that people will continue to explore how the development of "number sense" and other forms of mathematical reasoning may help us understand how children of different ages perceive and interpret quantitative maps.

In the meantime, studies that have already been done certainly raise interesting questions about the possible use of well-designed geography lessons to help enhance mathematical reasoning (Dorn et al. 2004; Pasnak et al. 2009).

In other words, if you want to raise math scores, you might have your students do some geography map activities! The research reviewed in this essay certainly suggests that there may be a good neurologic reason for trying that approach.

Overlaps Between Comparison and Other Modes of Spatial Thinking

As noted in the essay on Spatial Transitions, a line graph can be used to show the change in some condition, such as elevation, temperature, or house value, as you go from one place to another. Reading this kind of graph, however, can be described as like making a sequence of quantitative comparisons. Several studies have shown that both children and adults intuitively read the slope of a line as an indicator of rate (Gattis and Holyoak 1996; Gattis 2001; Tversky 2001; Gattis 2002). In this way, the distinction between mental comparison and the mental representation of spatial transitions can be blurred – the two activities both engage the intraparietal sulcus on the back side of the head; the primary difference lies in the fact that a spatial sequence or transition also engages parts of the frontal lobe in the brain to store and process information about the spatial ordering of observations (see the essay on spatial sequences for a list of references).

There is also some evidence of overlap between the process of comparison and the identification of similar positions on other continents or in other cities (spatial analogies). This overlap occurs because a quick estimation of relative position along a line or within an area seems to be subject to the same "misperception" as a quantitative comparison of sizes. Studies have shown that attempts to bisect a line have a leftward bias that parallels the non-linearity of the mental number line (Calabria and Rossetti 2005). This complication also appears when people compare color hues and gradients (Castelli et al 2006) or verbalized numbers with numerical quantifiers such as "at least three" (Clark and Grossman 2007; Gandini et al. 2008). These "higher-order" quantifiers, however, also "recruited right dorsolateral prefrontal cortex, suggesting involvement of executive resources like working memory" (McMillan et al. 2005). Indeed, adding tasks that put stress on working memory tended to reduce the leftward bias (Herrera et al 2008). Taken together with the gray-scale experiments described above, these findings show how comparisons of distance underlie some decisions about whether two locations are analogous, but thinking about spatial analogies involves a much more complex brain network. Incidentally, the complexity of these interactions between size, distance, brightness, and relative position may also help explain why people were unable to reconstruct a properly sequenced map legend for spectrally encoded two-variable maps (Olson 1981).

Finally, there are logical overlaps between spatial comparison and the recognition of spatial patterns and spatial associations on maps. These overlaps occur because people often describe spatial patterns or associations by using comparative language. For example, people might describe the spatial pattern of earthquakes along a particular fault as "aligned, like the earthquakes along the San Andreas fault, but in an east-west direction and over a much shorter distance than the San Andreas." Despite this use of similar vocabulary, we suggest that there is plenty of support for a logical distinction between the processes of comparing conditions at two specific places and comparing overall map patterns in two larger areas. When the first rigorously controlled fMRI studies of these two processes are done, we predict that the latter activities will engage far more areas of the brain, a tribute to their greater perceptual and logical complexity.

All in all, the first decade of the 21st century has witnessed a truly remarkable amassing of empirical research concerning the brain structures involved in qualitative and especially quantitative comparisons. The decade ended with a reminder that there is still more to learn; one of the newest studies in our review demonstrates a structural "dissociation" between numerical and logical quantifiers. This use of different brain structures may in turn be responsible for individual differences between people who compare magnitudes on a map by using numerical reasoning and those who compare through other logical mechanisms that are as yet poorly understood (Troiani et al. 2009; for an independent path to a similar conclusion about "marked individual differences," see Paulsen et al. 2008).

Issues with using a GIS to support thinking about spatial comparisons

The biggest issue in using a GIS to help compare places is the fact that a GIS display often hides key spatial aspects of the original data gathering and subsequent processing. Here is an all-too-common example: an attempt to use publicly available data to compare the fraction of personal income that is used for house payments in different neighborhoods. To do this, a researcher might

- find records of the prices of recently-sold houses in different neighborhoods.
- look at the census records to find the estimated household income in those neighborhoods.
- calculate some ratio of house price and household income.

The problem lies in the fact that a house sale price represents a single property, whereas the income figure is an average for an entire census tract. To the extent that all houses in a neighborhood are similar, this approach can provide reasonable estimates. If, however, there is great variation in house sizes and prices, the mismatch between the local scale of the house price and the broader areal base of the income data can produce unrealistic results. For one thing, it is not unreasonable to assume that the people who buy the more expensive houses in a neighborhood are also likely to have incomes that are higher than the neighborhood average.

This problem of spatial mismatch affects many topics, including

- mineral resources and standard of living. An almanac or online country guide can tell you what resources a country has within its borders; it can also tell you the average income of people in the country. Attempts to use resources as an explanation of income, however, seldom work well. The resources occur at specific locations, whereas the people may be living in many other places within the country, often far from any jobs in the mines or other influences from them.
- test scores in school districts and property value. Average reading scores are reported for an entire school district, but a typical school district includes a heterogeneous mix of local neighborhoods that may have people with different family sizes, income levels, and other variables that can have powerful influences on individual test scores.
- family income, educational level, or religious adherence and terrorist activity. The first three measures are usually reported as averages for entire countries or regions within countries, whereas terrorist hideouts tend to be highly localized and influenced by such things as terrain, ethnic heterogeneity, and the effectiveness of local police action.

All of these examples are variants of the so-called “ecological fallacy” – the tendency to assume that any individual physically located within a specific area on a map has all of the traits that are reported in summary statistics about that area.

Here is a familiar example to use in class: an occupational count shows that many people in a particular neighborhood work as domestic servants. Meanwhile, a census report shows a high average per-capita income in that census tract. The fallacy comes when someone puts those two kinds of data together and concludes that domestic servants have a high income (when in fact they are more likely to be low-paid employees of people with incomes that are high enough to bring the average way up). Economic comparisons of places are often flawed because occupation is an individual characteristic that is observed at a specific address, whereas income is an average value that is reported for a relatively large area.

In short, the safest comparison is between individual people, families, properties, or sample points in fields. Unfortunately, most publicly available data consists of averages or totals that are gathered from samples or larger areas. A GIS display can mask some key aspects of the input data. There is no solution to this problem that does not involve careful reading of the metadata and thorough understanding of the workings of the system!

REFERENCES ABOUT SPATIAL COMPARISON

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